R E P O R T R E S U M E S

ED 018 369 SE 004 140

MATHEMATICS PROGRAM FOR IDAHO PUBLIC SCHOOLS.

BY- ENGELKING, D.F.
IDAHO STATE DEPT. OF EDUCATION, BOISE

PUB DATE 65

EDRS PRICE MF-$0.25 HC-$1.52 36P.

DESCRIPTORS- *ARITHMETIC, *CURRICULUM GUIDES, *MATHEMATICS, *SECONDARY SCHOOL MATHEMATICS, ALGEBRA, ADVANCED PLACEMENT, CURRICULUM, ELEMENTARY SCHOOL MATHEMATICS, GEOMETRY, SPIRAL CURRICULUM, IDAHO STATE DEPARTMENT OF EDUCATION, BOISE, IDAHO,

THIS CURRICULUM GUIDE FOR ALL GRADES OF PUBLIC SCHOOL INSTRUCTION IS INTENDED TO ASSIST LOCAL SCHOOL DISTRICTS IN DESIGNING AN ADEQUATE MATHEMATICS PROGRAM THAT REFLECTS THE MODERN APPROACH. THE GUIDE PRESCRIBES A PROGRAM WHICH IS SUFFICIENTLY FLEXIBLE TO BE USED BY LARGE DISTRICTS HAVING A COMPLETE THREE-TRACK MATHEMATICS PROGRAM WITH FIVE YEARS OF SECONDARY MATHEMATICS AVAILABLE, AS WELL AS BY SMALL SCHOOLS OFFERING ONLY A MINIMUM PROGRAM. THE FOLLOWING INFORMATION IS INCLUDED IN THE MAIN BODY OF THE REPORT--(1) THE OUTLINE OF SCOPE AND SEQUENCE FOR THE ELEMENTARY MATHEMATICS PROGRAM, GRADES 1-6, (2) SUGGESTED CONTENT FOR THE PROGRAM, GRADES 1-8, AND (3) TITLE AND COURSE DESCRIPTION FOR THE MATHEMATICS OF THE SECONDARY CURRICULUM. IT IS HOPED THAT THESE GUIDES WILL BE OF SOME USE AT THE LOCAL LEVELS IN THE INITIATION OF PLANNING AND WRITING OF PROGRAMS THAT WILL ENSURE A PROGRESSIVE, ADEQUATE, AND CONTINUOUS PROGRAM RELEVANT TO THE NEEDS OF LOCAL STUDENTS. (RP)

U.S. DEPARTMENT OF HEALTH, EDUCATION & WELFARE
OFFICE OF EDUCATION

THIS DOCUMENT HAS BEEN REPRODUCED EXACTLY AS RECEIVED FROM THE PERSON OR ORGANIZATION ORIGINATING IT. POINTS OF VIEW OR OPINIONS STATED DO NOT NECESSARILY REPRESENT OFFICIAL OFFICE OF EDUCATION POSITION OR POLICY.

ED018369

MATHEMATICS PROGRAM

FOR

IDAHO PUBLIC SCHOOLS

D. F. ENGELKING

STATE SUPERINTENDENT OF PUBLIC INSTRUCTION

STATE OF IDAHO

DEPARTMENT OF EDUCATION

BOISE, IDAHO

SE 004 140

JAN 1 3 1967

State of Idaho

Department of Education

MATHEMATICS PROGRAM

for

IDAHO PUBLIC SCHOOLS

Published by
DIVISION OF INSTRUCTION
1965

D. F. Engelking
Superintendent
of Public Instruction

V. Reid Bishop
Deputy State Superintendent
Division of Instruction

TABLE OF CONTENTS

INTRODUCTION 1

ELEMENTARY SCHOOL MATHEMATICS 2

SECONDARY SCHOOL MATHEMATICS 3

TEXTBOOK ADOPTIONS, ELEMENTARY MATHEMATICS
BASIC TEXTBOOKS 4

TEXTBOOK ADOPTIONS, SECONDARY MATHEMATICS
BASIC TEXTBOOKS 4

ELEMENTARY MATHEMATICS, OUTLINE OF SCOPE AND SEQUENCE
GRADES ONE-SIX

- GRADE ONE 5
- GRADE TWO 5
- GRADE THREE 6
- GRADE FOUR 6
- GRADE FIVE 7
- GRADE SIX 7

SUGGESTED CONTENT FOR GRADE ONE 8

SUGGESTED CONTENT FOR GRADE TWO 11

SUGGESTED CONTENT FOR GRADE THREE 14

SUGGESTED CONTENT FOR GRADE FOUR 17

SUGGESTED CONTENT FOR GRADE FIVE 20

SUGGESTED CONTENT FOR GRADE SIX 23

SUGGESTED CONTENT FOR GRADE SEVEN 25

SUGGESTED CONTENT FOR GRADE EIGHT 28

SECONDARY MATHEMATICS CURRICULUM 31

TITLES AND COURSE DESCRIPTIONS 32

- FUNDAMENTAL MATHEMATICS I AND II, Grades 7-8
- GENERAL MATHEMATICS, Grades 9-10
- BUSINESS MATHEMATICS, Grades 9-10
- ALGEBRA I, Grades 8-12
- ALGEBRA II, Grades 9-12
- INTEGRATED GEOMETRY, Grades 9-12
- ADVANCED MATHEMATICS I, Grades 11-12
- ADVANCED MATHEMATICS II, Grade 12

INTRODUCTION

Recent changes in mathematics instruction have made it necessary for educators to make careful evaluation of the total mathematics program. Essentially, the changes that have taken place in the teaching of mathematics are in the approach rather than in the content. However, several new topics have been introduced. Changes made were designed to make mathematics more meaningful and to provide a strand of learning to carry students logically through all the mathematics studied. Movement away from rote memorization toward a true understanding of the meaning of mathematics will continue.

Mathematics, as the logical study of the properties of various sets of objects and the operations on these sets, involves computation. However, the development of high speed digital computers makes the development of skills in rapid computation unnecessary. This is not to say that a certain amount of computational skill is not necessary. Mathematics is a process as much as it is a product. It is a way of approaching a problem as well as the solution to a problem.

This guide attempts to reflect changes that have taken place in mathematics. It is hoped that it will assist local school districts in designing an adequate mathematics program. The guide is not meant to be prescriptive. The program is designed to be flexible and can be used for large districts having a complete three-track mathematics program with five years of secondary mathematics available, as well as by small schools offering only a minimal program.

A careful evaluation of the objectives of a school's program in mathematics, which includes teacher preparation, should be made before any selection of textbooks. It is essential that textbooks purchased for use with this guide reflect the latest approaches to the teaching of mathematics.

This brief guide is designed to aid in the initiation of local planning and writing. In the review of local problems and evaluation of curriculum objectives, cooperative efforts of individuals from all levels of mathematics education will help to ensure a progressive, adequate, and continuous program pursuant to the needs of local students.

ELEMENTARY SCHOOL MATHEMATICS

Changes in elementary mathematics have emphasized the need to substitute understanding and interest for rote performance. Instruction is enhanced when pupils are guided and facilitated in exploring, discovering and constructing concepts. The program in elementary mathematics should develop generalized concepts which may serve as tools for exploring mathematics further. To be able to manipulate numbers and symbols by rote learning of rules is not enough; pupils must understand concepts, principles, vocabulary, and structure. The school should provide pupils the opportunity to explore and discover for themselves patterns which exist--the goal should be to create a desire and skill in children which will cause them to think quantitatively. To realize this goal provision is made for sequential development in five areas:

1. Understanding number ideas
   - kinds of numbers - natural, rational, irrational, real
   - patterns
   - relationships of numbers and processes
   - basic principles for operations
   - symbols and forms for meanings and operations
2. Numeration systems - base ten - other bases
3. Efficient manipulation and use of symbols, facts, principles and processes
4. Proficient application of the number concepts and skills to problem solving
5. Habitual estimating and checking

The school's responsibility for attaining individual potential competence in these areas depends upon the teacher, the learner, the arithmetic content, and the instructional materials.

Effective teaching emphasizes arithmetic as a system of interrelated number ideas. In addition to the text and the accompanying manual, visual aids (chalkboards, manipulative devices, diagrams and number lines) are necessary to ensure more insight before new concepts are applied in practice in problem solving. Additional experiences when needed to establish any skill, fact, process, or application in problem solving, should follow meaningful development.

The grade level placement is not explicit. In some instances teachers may find it advantageous to present topics either earlier or later than suggested. Also it must be understood that topics presented in an earlier grade must be maintained and extended. Number concepts are presented with the spiral approach. They are introduced at one level and developed more completely at higher levels.

A wide range of ability, interest, and achievement exists in practically all classes. The individual development of all children is of major significance and must be continually considered in the organization of the class and provision of materials. The teacher should be sensitive to individual differences among students and should not assume that all students are prepared or qualified to study the same topic at the same time. Instead, the level of understanding and achievement of each child should be identified and he should be assisted to progress.

At each grade level materials are needed for a wider range than one level. For consistency and sequential continuity, the same basic text series should be used throughout the grades. Other series may serve as supplementary materials.

SECONDARY SCHOOL MATHEMATICS

The mathematics program of the junior high school is a transitional one. It builds on concepts taught earlier, and introduces new areas which will later be developed in the senior high school program.

As a developmental program, the scope and sequence includes the following:

- mathematical thinking
- increased mathematical concepts as youngsters are ready to begin more complex terms
- an opportunity should be given for pupils to develop mathematical generalizations
- the teaching of new concepts based on those previously learned

Throughout the junior high years the mathematics program attempts to integrate basic concepts found in the following areas:

- arithmetic
- algebra
- geometry
- structure and use of numeration systems
- fundamental operations
- equations, inequalities and measurement

Pupils vary greatly in abilities, interests, skills, and rates of achievement. Each pupil should be helped to develop his ability to the fullest.

Schools, making provision for varying rates of student ability and student achievement in the elementary grades, will find that some students have developed the

skills and understandings essential for successful completion of a course in Algebra I during the eighth grade year. Other students may not attain the required level of achievement until they have completed a course in general mathematics in the tenth grade. Therefore, there is a designed flexibility in the mathematics courses for secondary schools.

The minimal program in the mathematics scope and sequence provides two years of general mathematics. An accelerated program allows Algebra I to be taught in the eighth grade with a continuity of courses leading to Advanced Mathematics II (advanced placement mathematics) in grade 12. The scope and sequence also makes provision for a continuous mathematics program for all students in the secondary schools.

Secondary schools should develop a sequential mathematics program designed to meet the needs of their students. Course titles and course descriptions from the scope and sequence should be used where applicable.

TEXTBOOK ADOPTIONS
ELEMENTARY MATHEMATICS - BASIC TEXTBOOKS

Multiple basic textbook adoptions have been made for use in the elementary mathematics program. School districts should select textbooks for mathematics from the State adopted list.

Modern concepts included in the State adopted textbooks place emphasis upon utilizing individualized discovery techniques in building mathematical concepts. School districts that have had experience with modern mathematics programs should make their selection from the more modern textbook series. Districts without prior experience in modern mathematics should select a series and make provision for an in-service program specifically designed to acquaint the teachers with the selected series.

Elementary schools should select a single series to ensure a sequential mathematics program in grades 1-6. However, with careful planning, some series can be mixed and still provide for sequential mathematical development through the 8th grade.

SECONDARY MATHEMATICS - BASIC TEXTBOOKS

Multiple textbook adoptions have been made for secondary mathematics courses making provision for a flexible 3-track program. Students should be ready for modern concepts at the secondary level. If not, a transitional course should be provided to acquaint the students with modern concepts. Careful study of the total program should be made before textbooks for any one course are selected by a school district.

ELEMENTARY MATHEMATICS
OUTLINE OF SCOPE AND SEQUENCE
GRADES ONE-SIX

GRADE 1 - NUMBERS, NUMERATION, AND COMPUTATION: Understanding of base ten system through 99, place value grouping by ones and tens, fractional numbers and the numerals associated with 1/2, 1/3, 1/4, ordinal use of number. The structure and properties of addition and subtraction - basic facts sums and minuends of 6, 7, 8.

SETS: Understanding of one-to-one correspondence, equivalent and non-equivalent sets, set union and separation to introduce addition and subtraction, relation of the numeral 0 to the cardinal number of the empty set.

MEASUREMENT: Money - penny, nickel, dime; Linear measure - inch, foot; Liquid measure - cup, pint, quart; Time - hour, half hour, day, week, month.

GEOMETRY: Recognize simple geometric forms - line, circle, square, rectangle.

GRADE 2 - NUMBERS, NUMERATION, AND COMPUTATION: Natural numbers through 200. Place value, ones, tens, hundreds; Understanding of the structure and properties of addition and subtraction with computational skills extended to three digit numerals, with regrouping of tens and hundreds; Introduction of structure and properties of multiplication and division. (Computational skill to 5 x 5 optional.)

SETS: Review and expand equivalent and non-equivalent sets, subsets. Use set concept to define addition and multiplication and interpret subtraction and division.

MEASUREMENT: Money - understand value of all coins; Linear measure - foot, inch, yard; Liquid measure - cup, pint, quart, gallon; Weight - pound, 1/2 pound, 1/4 pound; Time - hour, days of week, months, seasons, use of calendar.

GEOMETRY: Recognize simple geometric forms - square, circle, rectangle, line, right angle, triangle.

GRADE 3 - NUMBERS, NUMERATION, AND COMPUTATION: Extend understanding of base ten to include four digits. Understanding of addition expanded by study of commutativity, associativity, and the additive property of zero. Subtraction considered as set separation and the inverse of addition. Division examined in terms of set partitioning, repeated subtraction, and the inverse of multiplication. Addition and subtraction of four digit numbers with and without regrouping. Development of the basic multiplication and division facts, with 2's, 3's, 4's, 5's, 6's, and 10's. Extend understandings of fractional numbers to include halves, fourths, thirds, fifths.

SETS: Use to define addition and multiplication and to interpret subtraction and division.

MEASUREMENT: Money - extend understanding of relative value of coins; Linear measure - half inch, inch, foot, yard; Liquid measure - cup, pint, quart, gallon, half-gallon; Weight - ounce, 1/4 pound, 1/2 pound, pound; Time - minute, 1/4 hour, 1/2 hour, hour, day, week, month; Temperature - degree Fahrenheit.

GEOMETRY: Engage in experiences for identifying simple geometric forms. Interpret and develop charts and graphs.

GRADE 4 - NUMBERS, NUMERATION, AND COMPUTATION: Natural numbers to include millions. Expand knowledge of place value of numerals. Expand addition and subtraction of natural numbers to include addition of four four place numbers and subtraction using four place numbers. Multiplication extended to include multiplication involving multipliers expressed by 1 and 2 figure numerals. Division extended to include division involving divisors expressed by 1 and 2 figure numerals. Recognize numerals associated with fractional numbers, 1/2, 1/3, 1/4, 1/5, 1/6, 1/7, 1/8. Develop understanding of equivalent fractions. Add and subtract like fractions. Expand Roman numerals through C.

SETS: - Review and expansion of equivalent and non-equivalent sets, and of the empty set. Explanation of addition as union of disjoint sets. Sets used to define multiplication and to interpret subtraction and division.

MEASUREMENT: Units of time from second to century. Understanding and computation practice with liquid and dry measures. Review of Fahrenheit thermometer scale. The arithmetic mean. Rounding and estimating numbers. Money as exchange for goods or money of equal value. Comparing and combining different units of linear measurement. Addition, subtraction, multiplication, and division computation in problem solving.

GEOMETRY: Introduction to nonmetric geometry of space; concepts of point, line, line segment, end points, angles, curves, polygons, circles.

GRADE 5 - NUMBERS, NUMERATION, AND COMPUTATION: Natural numbers maintained, (0-999,999); Base ten place value maintained; Roman numerals extended through M; Ordinals extended; Common fraction numerals maintained; decimal fraction numerals extended to tenths, hundredth; Addition, subtraction, multiplication and division of natural numbers maintained. Addition and subtraction of common and decimal fraction.

SETS: Review and extension of concept of sets to reinforce the understanding of basic computation skills and to review the concept of fractional numbers.

MEASUREMENT: Measurement equivalents and reduction of measures extended; Measurement of perimeters and areas of polygons; Review of liquid and dry measurement.

GEOMETRY: Study of nonmetric geometry of space; review and extend concepts of point, line, line segment, parallel lines, rays, angles, simple closed curves, polygons; Introducation of measurement of perimeter.

GRADE 6 - NUMBERS, NUMERATION, AND COMPUTATION: Natural numbers through billions; Base ten place value maintained and extended. Four processes using natural numbers maintained. Common fractions, addition and subtraction maintained, extended to include multiplication and division of fractions expressed by proper fraction numerals and mixed numerals; Fractions expressed by decimal fraction numerals (all processes); Use of ratios to express ordered pairs involving rate and comparison - fraction numerals as ratios; Introduction to percent.

SETS: A review of the concepts of sets including equality, equivalence, infinite sets, sets of numbers, proper and improper subsets, the operations of union and intersection, sets used to reinforce basic computational skills, addition in terms of union of disjoint sets, interpretation of multiplication using sets, and division interpreted by partitioning.

MEASUREMENT: Review and extend study of linear measurement, including the four arithmetic operations on measurement, techniques for measuring lengths and areas of simple closed curves.

GEOMETRY: Maintain concepts previously presented. Review of concepts of measurement of perimeter and area. An introduction to special nonmetric geometry, including prisms, cylinders and spheres. Exploration of volume.

SUGGESTED CONTENT FOR GRADE ONE

UNDERSTANDING NUMBER CONCEPTS

Before beginning the instructional program in arithmetic an evaluation of readiness should be made.

1. Participate in readiness activities - (concrete only).

2. Recognize groups to six.

3. Discover sets (one—to—one correspondence).

4. Understand concepts of number and the association of the number with the group or numeral.

5. Understand the place value of ones, tens.

6. Introduce ordinal idea - recognize ordinal names through tenth to clarify ideas of order and position.

7. Distinguish between even and odd numbers.

8. Understand the application of the number line.

9. Recognize zero as a symbol for "none," "not any," and as a place holder.

10. Discover commutative law and that subtraction is the inverse of addition (do not use terms commutative and inverse of addition with children).

11. Understand the use of more, more than, less, less than, take away, plus, equals, not equal and the symbols: $=$, $+$, $-$, $>$, $<$, $\neq$, and their use in mathematical sentences.

12. Learn two concepts of subtraction.

    a. to find how many are left

    b. to find how many are needed

13. Introduce the idea of multiplication and division (without symbols).

    a. understand that multiplication means putting together groups or sets of equal size

    b. use grouping as a basis for counting by ones, twos, fives, and tens

    c. understand that division means separating a group by repeated subtraction of groups of equal size

14. Measurement - compare and estimate size and distance.

15. Recognize the following geometric shapes: line, circle, square, triangle, rectangle.

16. Charts and graphs - develop the idea that a chart is a means of organizing and recording information and presenting ideas - graphs tell a number story.

APPLICATION OF NUMBER CONCEPTS, SKILLS, FORMS, AND VOCABULARY.

1. Read and write two digit numerals (numerals through 99).

2. Read and write number names through ten.

3. Add and subtract (with symbols). Use terms addends, sums, and missing addend often.

   a. combine groups through ten

   b. separate groups through ten

   c. rearrange groups through ten

4. Read and write addition and subtraction facts in both column and equation form (mathematical sentences).

5. Understand that common fractions are numbers (rational numbers).

   a. understand one-half, one-fourth and one-third of a whole object

6. Measurement - give emphasis that all measurement is approximate.

   a. learn terms of linear measurement

      1. inch, foot

   b. learn terms of liquid measure

      1. cup, pint, quart

   c. learn terms of weight

      1. pound

   d. learn terms for designating time

      1. hour, half hour, day, week, month

      2. read days of week and seasons

7. Money.

   a. recognize all coins

   b. understand relative value of one cent, five cents, ten cents

8. Quantity.

   a. understand pair, dozen

9. Thermometer

   a. explore thermometer (become aware of temperature and weather)

10. Problem solving.

   a. through oral expression formulate creative story problems - use dramatization, concrete objects and pictures in solving problems

   b. solve oral and later written problems involving the mathematical operations taught at this level

SUGGESTED CONTENT FOR GRADE TWO

UNDERSTANDING NUMBER CONCEPTS

1. Repeat and extend vocabulary and concepts studied in grade one.

2. Extend reading, writing, and meaning of number values through 200.

3. Extend ordinal idea.

4. Extend skill in distinguishing odd and even numbers.

   a. recognize that each even number can be separated into two equal groups - an even number is one that is evenly divisible by two

   b. recognize that there is always one left over when an odd number is separated into two equal groups

5. Understand place value - hundreds, tens, ones.

6. Strengthen understanding and meaning of zero.

7. Extend idea of mathematical sentences.

8. Learn concept of addition as combining groups (set union) and subtraction as the inverse of addition (set separation).

9. Develop the concepts of Commutative and Associative Laws of Addition (without using terms with children).

   a. a sum is unaffected by the order of the addend - 1+2=3, 2+1=3

   b. a sum is unaffected by the grouping of the addends - (1+(2+3)=6, (1+2)+3=6

10. Discover three major types of subtraction.

    a. how many are left

    b. compare two amounts

    c. how many more are needed

11. Comprehend simple column of three addends; sums of 18 for most pupils.

12. Extend readiness for multiplication and division.

    a. extend the concept of multiplication by combining equal groups (two 4's, two 5's, two 3's, two 6's, two 7's, two 8's, two 9's)

    b. extend the concept of division by separating a group to formulate equal parts

13. Extend understanding of equivalent values of money and measures.

14. Extend the concept of order.

    a. develop the understanding of grouping and regrouping

    b. develop the understanding of order of decimal

    c. provide readiness for concepts of ratio by:

        1. identifying equal quantities of similar objects

        2. distributing equal quantities of objects to equal groups

APPLICATION OF NUMBER CONCEPTS, SKILLS, FORMS, AND VOCABULARY

1. Read and write number symbols to 200 or more.

2. Understand and use symbols: $=, -, +, >, <, \{\}, \neq$

3. Read number words through twenty.

4. Review and extend experiences by counting to 200 by 2's, 10's, 5's.

5. Review and reinforce the work of addition as outlined in the first grade emphasizing:

    a. adding is counting on or moving toward the right on a number line

    b. subtracting is the inverse or undoing of adding or moving toward the left on the number line

6. Work toward mastery of easy basic facts through sums and minuends of 9, optional through 18.

7. Extend addition of one, two, and three digit numbers - regroupings of hundreds, tens, i.e. 100+10+1=111

8. Extend subtraction of one, two and three digit numbers - (with regroupings of hundreds, tens, and ones, i.e. 211-100-11=100).

9. Common fractions.

    a. renew and extend the meaning of 1/2, 1/4, 1/3 (one-half of a group, one-half, one-fourth, and one-third of an object).

10. Maintain and extend the following concepts of measurement:

    a. linear - foot, inch, yard

    b. liquid - cup, pint, quart, gallon

    c. weight - pound, 1/2 pound, 1/4 pound

d. time - 1/2 hour, days of week, months of year, seasons, use of calendar

e. money -

1. understand value of penny, nickel, dime, quarter, half dollar and dollar

2. read money amounts with ¢ and $ signs

f. temperature - begin temperature reading

11. Recognize the simple geometric forms as:

a. square, circle, rectangle, straight line, right angle, triangle

b. develop the habit of calling geometric forms by their geometric names

12. Charts and graphs.

a. practice making very simple graphs about daily experiences

b. develop an awareness of the function of a chart as a way of recording information for future use

13. Problem solving.

a. use many oral word problems to provide a setting for the study of

1. addition

2. subtraction

3. multiplication (repeated addition)

4. division (separating to form equal groups)

b. learn to read and work problems by using several methods of arriving at solutions

c. read and interpret three steps in problem solving

1. word symbols

2. representative material

3. number symbols

d. dramatize story problems

e. give children the opportunity to compose problems about money, measurements, calendar

f. encourage mental solutions without written computation

SUGGESTED CONTENT FOR GRADE THREE

UNDERSTANDING NUMBER CONCEPTS

1. Review of material presented previously. Careful evaluation of readiness.

2. Extend understanding of kinds of numbers and concept of numbers, whole numbers, counting numbers, even numbers, odd numbers, rational numbers.

3. Place value.

   a. extend understanding of base ten in value through thousands

   b. reinforce understanding of zero as a digit meaning "not any" and serving as a place holder

   c. regroup numbers through the hundreds place

4. Review language of sets.

5. Extend and reinforce addition and subtraction ideas.

   a. use of zero in addition to and subtraction from a number with the original number as the answer (3+0=3 and 3-0=3)

   b. subtraction with zero as the answer (4-4=0)

   c. read and write addition facts in both number sentences and columns (100+10+1=; 100)
      10
      1

   d. Cummutative Law in Addition (changing the order of the addends does not affect the sum)

   e. Associative Law in Addition (addends may be grouped in any way without affecting the sum)

   f. subtraction is the inverse of addition

   g. use of the number line for addition and subtraction

   h. recognition and solution of three different types of subtraction situations:

      1. find out how many are left

      2. compare two amounts

      3. find out how many more are needed

6. Understand relation between basic addition facts and higher decade addition, with and without regrouping.

7. Add columns of one and two place numbers with four addends and three digit numbers with three addends.

8. Subtract two and three digit numbers with and without regrouping.

9. Understand that multiplication is a short method of adding like groups.

10. Understand that factors may be interchanged without affecting the product (multiplication is commutative).

11. Discover the generalization that whenever zero is a factor the product is zero.

12. Discover the generalization that any number multiplied by 1 results in the original number.

13. Extend the concept that division is the inverse of multiplication.

14. Discover the generalizations:

    a. any number divided by 1 results in the original number

    b. any number divided by itself results in 1

    c. zero divided by any number results in zero

15. Understand that division is used to find:

    a. the number of equal groups (measurement or comparison)

    b. the size of the equal groups (partition)

16. Develop basic multiplication facts with 2's, 3's, 4's, 5's, 6's, and 10's.

17. Develop division facts: 2's, 3's, 4's, 5's, 6's, and 10's.

18. Idea of equivalence (the child discovers that a natural number can be designated by many different symbols as (5+1), (1+5), (4+2), (2+4) and (3+3) all belong to the same equivalence class 6.

19. Continue estimations and calculations.

APPLICATION OF NUMBER CONCEPTS, SKILLS, FORMS, AND VOCABULARY

1. Reading and writing four digit numerals.

2. Read number names through one hundred.

3. Roman numerals through XII (optional further).

4. Strive for mastery of basic multiplication facts.

5. Strive for mastery of division facts.

6. Use the number line to divide a number by moving to the left from the number in equal steps.

7. Fractions - 1/2, 1/3, 1/4, 1/5, - discover that dividing by abstract numbers is another way to find equal parts of a whole object or groups.

8. Decimals - read and write dollars and cents.

9. Maintain and extend the following concepts of measurement:

   a. linear - half inch, inch, foot, yard

   b. liquid - pint, quart, cup, gallon, half gallon

   c. weight - ounce, 1/4 pound, 1/2 pound, pound

   d. time - day, week, month, hour, minute, quarter hour , half hour

   e. temperature - degree Fahrenheit

   f. money

      1. understand the use of the decimal point in writing numerals that represent money

      2. extend the understanding of the relative value of coins (make change to one dollar)

10. Engage in experiences for identifying simple geometric forms as: circle, square, rectangle, triangle, point, line segment, angle, quadrilateral.

11. Interpret and develop charts and graphs.

   a. simple bar graphs

   b. picture charts of various types

   c. simple maps of neighborhood, school, or classroom

   d. the array pattern for multiplication

12. Solve problems.

   a. solve simple problems involving one and two processes

   b. create problems from realistic situations

   c. estimate solutions of problems without computation

   d. use many oral word problems

SUGGESTED CONTENT FOR GRADE FOUR

UNDERSTANDING NUMBER CONCEPTS

1. Evaluate readiness to begin content of fourth grade level.
2. Read and write 9 digit numerals.
3. Continue multiplication and division facts in related groups through 81.
4. Review and extend the concept of place value and regrouping.
5. Review of set vocabulary, idea of set, sets, sub sets, members of sets, finite sets, empty set, likenesses, differences, less than, more than, same as, equal, one-to-one correspondence, odd, even, and compare.
6. Use odd and even numbers.
7. Extend ordinal concepts.
8. Read and write Roman numerals through C, and recognize D and M.
9. Extend understanding of Commutative and Associative Laws of Addition.
10. Extend understanding of the three types of subtraction problems.
    a. additive - how many more are needed
    b. subtractive - how many or how much is left
    c. comparative - the difference between two amounts
11. Develop ability to round off numbers and estimate answers.
    a. use number lines
12. Increase understanding of relationship of multiplication to addition.
13. Strengthen understanding of commutative and associative properties of multiplication (commutative - $8x7=7x8$, associative - $(6x5)x8=6x(5x8)$.
14. Begin practice in the factoring of whole numbers.
15. Strengthen understanding of generalization concerning zero and one. ($0xn=0$, $nx0=0$, $1xn=n$, $nx1=n$)
16. Introduce concept of prime and composite numbers.
17. Increase understanding of relationships of division to subtraction (show that division is a short cut method of subtracting equal groups.)
18. Review generalizations concerning one and zero ($n \div 1=n$, $n \div n=1$).

19. Extend understanding of multiplying involving regrouping and zero.

20. Extend dividing by one-digit number with and without remainders and two digit divisors ending in zero.

21. Increase skill in multiplying and dividing as inverse processes, one used to verify the other.

22. Understand division process through the use of partial factors.

23. Understand the concept of averages (arithmetical mean).

24. Estimate answers using rounded numbers.

25. Introduce addition and subtraction of like fractions.

APPLICATION OF NUMBER CONCEPTS, SKILLS, FORMS, AND VOCABULARY

1. Add 2, 3, and 4 place numbers regrouping to tens, hundreds, and thousands.

2. Extend column addition to 5 one place numbers and 4 two, three, and four place numbers.

3. Increase skill in addition and subtraction through the use of mathematical sentences.

4. Subtract four place numbers regrouping to four places (use expanded notation).

5. Attain proficiency in adding and subtracting as inverse processes. Use number line to review inverse relationships.

6. Increase skill in use of zero in addition and subtraction.

7. Continue practice in methods of checking addition and subtraction.

8. Review multiplication and related division facts.

9. Multiplication of two digit and three digit numbers.

10. Find averages (arithmetical mean).

11. Fractions, use number line and diagrams.

    a. recognize 1/2, 1/3, 1/4, 1/5, 1/6, 1/7, 1/8 of the whole object and of the group and show that fractions may have different number names as 4/6=2/3, 2/4=1/2, 4/8=1/2

    b. add and subtract like fractions

12. Decimals

    a. continue using decimal point in money. Use fundamental computational operations in dealing with money

13. Measurement.

    a. recognize dozen, inch, foot, yard, pint, quart, gallon, peck, bushel, ounce, pound, week, month, year

        1. change to higher or lower units of measure

    b. read and write hours, seconds, minutes, a.m., p.m., noon, midnight

        1. change to higher or lower units of measure

    c. study time schedules for school day, airplane, train, bus, radio and TV network

    d. read thermometers, read and write temperature

    e. count money and make change

14. Engage in experiences to maintain and extend the ability to recognize and reproduce simple geometric forms.

15. Charts and graphs.

    a. make and interpret simple graphs and charts

    b. extend ability to draw and interpret maps

16. Problem solving.

    a. solve simple problems involving one or two operations using the four fundamental processes

    b. extend understanding of the steps in problem solving

SUGGESTED CONTENT FOR GRADE FIVE

UNDERSTANDING NUMBER CONCEPTS

1. Evaluate readiness to begin content of fifth grade.
2. Extend concept of ordinals through hundredth.
3. Extend concept of Roman numerals through M.
4. Add and subtract through five place numerals.
5. Continue multiplying by three place factors (optional four place).
6. Extend dividing with two-place divisors.
7. Review and expand the meanings of fractions.
   a. understand fractional parts of a whole object
   b. compare fractional parts
   c. find fractional part of a group
   d. add and subtract like and unlike fractions
   e. change improper fractions to equivalents of whole numbers plus fractions
   f. add whole numbers and fractions
   g. write remainders as fractions when reasonable
8. Decimal fractions.
   a. interpret decimal fractions as an extension of our number system
      1. use number line
   b. express tenths as common fractions
   c. express hundredths as common fractions
   d. express money as hundredths
   e. write mixed fractions using decimal notation
   f. add and subtract tenths and hundredths
   g. multiplication of decimals - using money concept
   h. division of decimals - using money concept

9. Ratio.

   a. use fractions as a means of comparison

   b. introduce rate pairs

10. Geometric concepts.

   a. maintain and use simple geometric forms

   b. develop concept of areas and perimeters (parallelograms)

   c. introduce ordered pairs

   d. recognize types of angles - acute, obtuse, right

   e. recognize types of triangles - right, isoceles, equilateral, scalene

   f. recognize and use terms - altitude, congruent, perpendicular

APPLICATION OF NUMBER CONCEPTS, SKILLS, FORMS, AND VOCABULARY

1. Review basic facts in addition, subtraction, multiplication and division.

2. Review regrouping process.

3. Review and extend understanding of sets.

4. Use the number line extensively.

5. Review and extend measurement concepts.

   a. standard units of measure (English)

      1. review concepts and skills previously learned

      2. convert units of measure to larger or smaller units

      3. learn the meaning of square measure, perimeters (area of parallelograms)

      4. use liquid and dry measure as volume

6. Fractions - add and subtract like and unlike fractions and mixed numbers.

7. Read, interpret, and make simple picture, bar, line, and circle graphs.

8. Interpret maps and globes (longitude, latitude, scales, time zones).

9. Read and interpret simple tables.

10. Solve problems.

a. emphasize steps in problem solving (writing mathematical sentences in problem solving)

b. solve multiple process problems

c. solve problems of various types

11. Review and extend geometric concepts.

a. find areas and perimeter of parallelograms

SUGGESTED CONTENT FOR GRADE SIX

UNDERSTANDING NUMBER CONCEPTS

1. Evaluate readiness to begin content of sixth grade.
2. Read and write numerals through billions.
3. Extend the base-ten numeration system through the decimal system of notation.
4. Read and write Roman numerals with facility.
5. Extend understanding of Associative and Commutative Laws (addition).
6. Review and extend concept of sets introduced in previous grades.
7. Multiplication and division.
   a. master multiplying and dividing by powers of ten
   b. divide by a three digit divisor
   c. understand more fully commutative, associative, and distributive law (multiplication)
8. Fractions (common)
   a. add and subtract fractions
   b. multiply and divide fractions
9. Decimal fractions.
   a. read, write, and use decimal fractions through thousandths
10. Understand relationship and conversion of equivalent percents, decimals, and fractions.
11. Ratio.
    a. use ratio in solving problems that involve the idea of rate or comparison
12. Per cent.
    a. introduce the concept of per cent - recognize symbol
13. Understanding uses of exponents and powers in base ten.

14. Extend and develop measurement concepts.

    a. standard units of measure (English)

        1. review concepts and skills previously learned

        2. introduce the idea of cubic measure

        3. add and subtract measures

        4. apply the principle of regrouping in multiplying and dividing measures

15. Geometry.

    a. extend interpretation of latitude and longitude; know the meaning of meridian

    b. recognize cubes, cones, cylinders, and spheres as solids

APPLICATION OF NUMBER CONCEPTS, SKILLS, FORMS, AND VOCABULARY

1. Compute with speed and accuracy.

2. Calculate mentally sums and differences.

3. Estimate results of operations.

4. Graphs, charts, and scales.

    a. maintain and extend skill in reading, interpreting, and drawing graphs

    b. draw to scale

    c. chart various experiences

    d. read scales of various maps

5. Solve problems.

    a. understand processes for problem solving

    b. solve problems of various types involving practical applications

    c. solve problems involving numbers expressed as fractions

6. Review and extend geometric concepts

    a. solve problems involving perimeters and areas of rectangles

    b. solve problems involving volume of rectangular solids

SUGGESTED CONTENT FOR GRADE SEVEN

UNDERSTANDING NUMBER CONCEPTS

1. Review of number meanings and concepts presented previously.

2. Extend the concepts of sets, complement set, subsets, and set notation to include $\emptyset$ (null set), U (universe), $\subseteq$ (subset), $\notin$ (not an element), $\cup$ (union), and $\cap$ (intersection).

3. Develop the concept of pairs, number pairs, ordered pairs, and Cartesian set.

4. Extend the development of understandings of mathematical sentences, closed sentences, open sentences, equations, and inequalities.

5. Expand the knowledge of numeration systems.

   a. history of early numeration systems

   b. bases less than ten

   c. decimal system

   d. place value and symbols

   e. approximate and rounded numbers

   f. primes and composite numbers

6. Introduce exponents and roots.

7. Continue development of concepts of number relationships.

   a. order of operations

   b. properties of natural numbers

      1. commutative property for addition

      2. commutative property for multiplication

      3. associative property for addition

      4. associative property for multiplication

      5. distributive property for multiplication and addition

   c. property of closure

8. Review and extend the fundamental operations with natural numbers.

   a. addition of natural numbers

b. multiplication of natural numbers

c. inverse operations

1. subtraction

2. division

9. Develop further understandings of fractions.

a. equivalent fractions

b. reduction to basic fractions

c. least common denominator

d. fundamental operations with common fractions

e. converting common fractions to decimal fractions and the reverse

f. fundamental operations with decimal fractions

g. ratio

10. Extend the concepts of per cent and percentage.

a. meaning of per cent

b. relationships among common fractions, decimal fractions, and per cents

c. unifying concepts involved in finding base, rate, and percentage

d. per cents greater than 100% or less than 1%

11. Increase the understanding of geometric shapes and their properties.

a. lines, segments, rays, and planes

b. angles, circles, and polygons

12. Extend understanding of measurement.

a. formulas

1. area

2. weight

b. metric system

APPLICATION OF NUMBER CONCEPTS, SKILLS, FORMS, AND VOCABULARY

1. Use many oral and written problems.

2. Explore the history of early numeration systems and explore bases other than ten.

3. Maintain and extend fundamental operations of whole numbers and fractions.

   a. the commutative, associative, distributive, and closure properties

   b. new methods of computation and systems of checking

   c. inverse operations

4. Continue developing understandings of fractions (equivalent fractions, lowest common denominator, addition, subtraction, multiplication, and division).

5. The construction and interpretation of natural number line, broken line, bar and circle graphs.

6. Extend the concept of per cent and percentage (meanings and application).

7. Continue the development of geometric shapes and their properties and emphasize lines, segments, rays, planes, angles, circles and polygons.

8. Develop the understanding of graphs and statistics.

   a. natural number line

   b. pictographs, line, bar, and circle graphs

   c. frequency tables and histographs

9. Use of measurement in determining:

   a. weight

   b. length, perimeter, area, circumference, and volume

   c. time, speed

   d. capacity (liquid and dry)

   e. angles

SUGGESTED CONTENT FOR GRADE EIGHT

UNDERSTANDING NUMBER CONCEPTS

1. Understand the structure and use of our numeration system.
   a. review and expand the knowledge of the decimal system
   b. extend knowledge and use of exponents and roots
   c. rationals, irrationals, real numbers
   d. number pairs and sets
   e. primes and composites
2. Augment number relationships.
   a. properties of equality - transitive, reflexive, symmetric
   b. equations
   c. commutative, associative, distributive, and closure properties
   d. algebraic inequalities
3. Review fundamental operations with whole numbers.
4. Continue fundamental operations using common and decimal fractions.
5. Expand the concepts of per cent and percentage.
   a. Per cents greater than 100%
   b. Per cents less than 1%
6. Enlarge the concepts of geometric shapes and their properties.
   a. rays, lines, segments, and angles
   b. triangles
   c. symmetry
   d. quadrilaterals, circles, polygons
7. Increase the knowledge of measurement.
   a. history
   b. metric system

c. area

1. rectangle

2. square

3. parallelogram

4. trapezoid

5. circle

6. triangle

7. pyramids

8. rectangular solids

9. cylinder

10. cone

11. spheres

d. volume

1. rectangular solids

2. pyramids

3. cones and cylinders

4. spheres

e. time zones and longitude

f. utility measurements (gas, water, electricity)

g. temperature: Fahrenheit and Centigrade

8. Introduce the concept of median and mode.

APPLICATION OF NUMBER CONCEPTS, SKILLS, FORMS, AND VOCABULARY

1. Use many oral and written problems.

2. Understand the structure and use of our numeration system.

3. Number relationships

a. properties of equality

b. equations

c. commutative, associative, distributive, and closure properties

d. algebraic inequalities

4. Continue fundamental operations with whole numbers.

5. Continue fundamental operations of fractions.

   a. ratio

   b. proportion

6. Continue the concepts of per cent and percentage. Emphasize applications of:

   a. interest

   b. discount

   c. per cents greater than 100% and less than 1%

7. Extend the knowledge of geometric shapes and their properties.

   a. rays, lines, segments

   b. angles, triangles, quadrilaterals, circles, and polygons

8. Further develop the use of measurement and its application to area and volume.

9. Develop the understanding of graphs and statistics.

   a. making and reading bar, line, and circle graphs

   b. coordinates, distances, plotting points, graphs in a plane

   c. histograms and frequency distributions

   d. mean, median, and mode

SECONDARY MATHEMATICS CURRICULUM

	Level I	Level II	Level III
7th Grade	Fundamental Math I	Fundamental Math I	Fundamental Math (accelerated)
8th Grade	Fundamental Math II	Fundamental Math II	Algebra I
9th Grade	General Mathematics	Algebra I	Integrated Geometry or Algebra II
10th Grade	Business Mathematics	Integrated Geometry or Algebra II	Integrated Geometry or Algebra II
11th Grade	Algebra I	Integrated Geometry or Algebra II	Advanced Math I (analysis - trigonometry - introduction to calculus)
12th Grade	Integrated Geometry	Advanced Math I	Advanced Math II (advanced placement mathematics)

TITLES AND COURSE DESCRIPTIONS

FUNDAMENTAL MATHEMATICS I AND II - Grades 7 - 8

These courses are designed for students at the 7th and 8th grade levels. They review the mathematics taught in the earlier grades at a mature level and include an introduction to algebra and geometry. Accelerated students who are to be programmed for Algebra I in grade 8 should devote most of their 7th grade year to Fundamental Mathematics II. Course content for Fundamental Mathematics is found on pages 25 to 31.

GENERAL MATHEMATICS - Grades 9 - 10

General Mathematics is designed for students who need further development of their mathematical abilities and understandings to be able to solve problems of everyday living. It includes topics from arithmetic, algebra, and geometry. Experiences provided include work with the following: whole numbers, fractions, decimal fractions, circles, triangles, percentage, elementary algebra, area, volume, ratio, and graphs. For some students this may be a terminal course. Other students may take this course as a prerequisite to Algebra I.

BUSINESS MATHEMATICS - Grades 9 - 10

This course is designed to provide arithmetic experiences of a practical nature that arise in the everyday life of all people. It includes instruction in computing percentage, discounts, mark-up, interest and dividends. Experience is provided in graphing, charting, record keeping, and practical measurement. Emphasis is given to improving skills in the fundamental operations.

ALGEBRA I - Grades 8 - 12

The course must be treated as a study of mathematical structure from an algebraic point of view, of the real number system and its use in the techniques of algebra. The course should include traditional topics, including quadratic equations, ratio and variations, and inequalities and functions. Students should understand mathematical definitions and the nature of proof, as well as gain confidence in the use of mathematical language.

ALGEBRA II - Grades 9 - 12

This course should teach algebra as a logical, meaningful structure, based on a few axioms from which other statements are proven. The course should review the properties of the number system, the basic algebraic processes, and the notion of set and set language in order to teach the concept of function as expressed in set terminology. Additional topics should include exponents including negative and fractional numbers, the pure imaginaries, and complex numbers, through polynominal, exponential and logarithmic functions.

INTEGRATED GEOMETRY - Grades 9 - 12

Integrated Geometry includes an understanding of proof as an instance of deductive reasoning. The course includes the topics of congruence, geometric inequalities, parallelism and similarity. Changes in this course consider coordinate and solid geometry with topics on locus and numerical trigonometry. If time permits, topology, projective geometry and vectors should be included.

ADVANCED MATHEMATICS I - Grades 11 - 12

This fourth year of mathematics will vary, depending upon local conditions, but should consist mainly of topics from trigonometry, analytic geometry, and advanced topics in high school algebra. This course could be called a Calculus Preparatory Course.

ADVANCED MATHEMATICS II - Grade 12

A fifth year of mathematics is recommended as an honor or advanced placement course. This course should be developed in co-operation with a local university, or through the State Department of Education.